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Reg. No. : ....

Name : ....

First Semester B.C.A. Degree Examination, January 2013
(Career Related First Degree Programme Under CBCSS)
(Group 2(b))
Complementary Course – I
MM 1131.9 : MATHEMATICS – I

Time : 3 Hours Max. Weightage : 30

All the first **16** questions are **compulsory**. They carry **4** weightages in all.

1. The value of sinh 0 is

2. If the value of sinh x is $-\frac{3}{4}$, then the value of cosh x is

3. Find $\frac{dy}{dx}$, where $y = x \sinh x - \cosh x$.

4. Examine whether Rolle's theorem can be applied to the function $f(x) = \tan x$ for the interval $[0, \pi]$.

5. General solution of the differential equation $\frac{dy}{dx} = \sin x$ is

6. A curve is defined by the condition that at each of its points (x, y), its slope is equal to the square of the abscissa of the point. Express this in terms of a differential equation.

7. Solve the differential equation $\frac{dy}{dx} + 2xy = 0$.

8. When we say that an integer is a common divisor of other two integers ?

P.T.O.

9. If gcd (a, d) = d, then gcd $\left(\frac{a}{d}, \frac{b}{d}\right)$ =

10. State True/False : If $a|c$ and $b|c$ with gcd (a, b) > 1, then $ab|c$.

11. Express sin 6θ in terms of powers of sin θ and cos θ.

12. Find the principle amplitude of the complex number 1.

13. The value of $|e^{i\theta}|$ is

14. State Dirichlet's conditions.

15. Identify the objective function and constraints in the following optimization problem :

Maximize

$$z = 3x_1 + 4x_2$$

Subject to the constraints

$$4x_1 + 2x_2 \leq 80$$
$$2x_1 + 5x_2 \leq 180$$
$$x_1, x_2 \geq 0$$

16. Express the following linear programming problem in the standard form :

Maximize

$$z = 7x_1 + 5x_2$$

Subject to the constraints

$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

Answer **any 8** questions from among the questions **17** to **28**. They carry **1** weight each.

17. Prove that $\cosh^2 \frac{x}{2} = \frac{1}{2}(\cosh x + 1)$.

18. Verify Lagrange's mean value theorem for the function $f(x) = e^x$ on [0, 1].

19. Find the general solution of the differential equation $y'' - y' - 2y = 0$.

20. Verify that the function $u = x^2 - y^2$ is a solution of the two dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

21. Find $\mathcal{L}^{-1}\left[\frac{s+1}{s^2+1}\right]$.

22. Find the g.c.d. of – 12 and 30 and express it as a linear combination of two integers.

23. Show that 41 divides $2^{20} - 1$.

24. Calculate the value $\phi(360)$, where ϕ is the Euler phi-function.

25. Separate into real and imaginary parts the expression $\sin(x + iy)$.

26. If $\sin(A + iB) = x + iy$, show that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$.

27. Find all basic solutions of $x_1 + x_2 + 3x_3 = 10$ and $2x_1 + 3x_2 + x_3 = 15$.

28. Find the Fourier series of f given by

$$f(x) = \begin{cases} -k, & \text{when } -\pi < x < 0 \\ k, & \text{when } 0 < x < \pi \end{cases} \text{ and } f(x + 2\pi) = f(x).$$

Answer **any 5** questions from among the questions **29** to **36**. They carry **2** weights each.

29. Find the differential coefficient of $(\sin x)^x + \cos x^{\tan x}$ with respect to x.

30. If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)\, y_{n+2} + 2xy_{n+1} - n(n + 1)\, y_n = 0$.

31. Solve the differential equation $10x^2 \frac{d^2y}{dx^2} + 46x\frac{dy}{dx} + 32.4y = 0$.

32. Solve $xp + yq = 3z$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

33. Find the remainder obtained when 5^{38} is divided by 11.

34. a) State Wilson's theorem.

    b) Prove that $5^{2n+2} - 24n - 25$ is divisible by 576.

35. Find all values of $(-8i)^{\frac{1}{3}}$.

36. Find the Fourier series of f given by

    $f(x) = x$,

    where $-\pi < x < \pi$ and $f(x) = f(x + 2\pi)\ \forall x \in \mathbf{R}$.

Answer **any 2** questions from among the questions **37** to **39**. These questions carry **4** weights **each.**

37. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

38. a) If p is a prime, then prove that $a^p \equiv a \pmod p$ for any integer a.

    b) Show that $n^7 - n$ is divisible by 42.

39. Solve the following problem graphically :

    Maximize

    $$z = 3x_1 + 4x_2$$

    subject to the constraints

    $$4x_1 + 2x_2 \leq 80$$
    $$2x_1 + 5x_2 \leq 180$$
    $$x_1, x_2 \geq 0$$